Guidelines for Writing Elemental Proofs in SET THEORY AND OTHER NOTES

<u>Definition</u>: A set A is <u>non-empty</u> or <u>$A \neq \emptyset$</u>

$\Longleftrightarrow$ There exists an element $x \in \mathcal{U}$ such that $x \in A$.

<u>Guidelines for Writing Elemental Proofs</u>

① $\therefore x \in A \cap B$ [THIS IS FOLLOWED BY:]

$\therefore x \in A$ AND $x \in B$ by definition of "Intersection"

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② $\therefore x \in A \cup B$ [THIS IS FOLLOWED BY:]

$\therefore x \in A$ OR $x \in B$ by definition of "Union"

CASE 1 ($x \in A$)

... $\therefore$ ~~ IN CASE 1.

CASE 2 ($x \in B$)

... $\therefore$ ~~ IN CASE 2.

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③ $\therefore C \cap D \neq \emptyset$ [IS FOLLOWED BY:]

$\therefore$ There exists an element $x \in \mathcal{U}$ such that $x \in C \cap D$. [see ① Next]

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④ PROVE "$A \cap B = \emptyset$" using a Proof-by-Contradiction.

Guidelines (Continued)

(5) To Prove "$x \in A \cup B$":

[FIRST PROVE $x \in A$]
$\therefore x \in A$
$\therefore$ $x \in A$ OR $x \in B$ by Generalization
$\therefore$ $x \in A \cup B$ by definition of "UNION"

OR

[FIRST PROVE $x \in B$]
$\therefore x \in B$
$\therefore$ $x \in A$ OR $x \in B$ by...
$\therefore$ $x \in A \cup B$ by...

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(6) To PROVE "$x \in A \cap B$"

[FIRST, PROVE $x \in A$] $\therefore x \in A$
[NEXT, PROVE $x \in B$] $\therefore x \in B$
$\therefore$ $x \in A$ AND $x \in B$ by Conjunction
$\therefore$ $x \in A \cap B$ by definition of "Intersection"

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(7) To PROVE "$x \in A - B$"

[FIRST, PROVE $x \in A$] $\therefore x \in A$.
[NEXT, PROVE $x \notin B$] $\therefore x \notin B$.
$\therefore$ $x \in A$ AND $x \notin B$ by Conjunction.
$\therefore$ $x \in A - B$ by definition of Set Difference.

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(8) $\therefore$ $x \in C$ AND $x \in D$ [IS FOLLOWED BY:]
$\therefore$ $x \in C$ by Specialization. [THAT IS, IF YOU NEED THIS FACT]
$\therefore$ $x \in D$ by Specialization [THAT IS, IF you need this FACT]

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(9) $\therefore$ $x \in B^c$ [IS FOLLOWED BY:]
$\therefore$ $x \notin B$ by definition of "Complement"

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(10) $\therefore$ $x \notin A \cap B$ [IS FOLLOWED BY:]
$\therefore$ It is FALSE THAT $x \in A \cap B$.
$\therefore$ It is FALSE THAT $x \in A$ AND $x \in B$ by defn of "$\cap$".
$\therefore$ $x \notin A$ OR $x \notin B$ By De Morgan's LAWS OF LOGIC.

PROOF FRAGMENTS SHOWING THE APPLICATIONS OF UNIVERSAL MODUS PONENS AND UNIVERSAL MODUS TOLLENS IN WRITING ELEMENTAL PROOFS IN SET THEORY:

PROOF FRAGMENT	DEFINITION
Suppose $A \subseteq B$.	For All $x \in U$, If $x \in A$, then $x \in B$.
Suppose $t \in A$.	$t \in U$ AND $t \in A$.
$\therefore t \in B$ by UNIVERSAL MODUS PONENS	$\therefore t \in B$ by UNIVERSAL MODUS PONENS.

PROOF FRAGMENT	DEFINITION
Suppose $A \subseteq B$.	FOR ALL $x \in U$, If $x \in A$, Then $x \in B$.
Suppose $t \notin B$	$t \in U$ AND $t \notin B$.
$\therefore t \notin A$ by UNIVERSAL MODUS TOLLENS	$\therefore t \notin A$ by UNIVERSAL MODUS TOLLENS